hw set 2
sect 5.1
p 352

1. a) lower Estimate = L_5

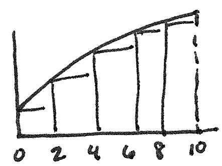
$\Delta x = 2$

$L_5 = f(0)\Delta x + f(2)\Delta x + f(4)\Delta x + f(6)\Delta x + f(8)\Delta x$

$= 2(1 + 3 + 4\frac{1}{3} + 5\frac{1}{3} + 6\frac{1}{4})$

$= (19\frac{11}{12})\cdot 2 = 39\frac{5}{6}$

$= 39.8\overline{3}$



Upper Estimate = U_5

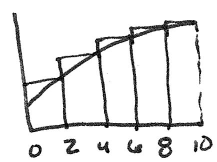
$\Delta x = 2$

$U_5 = \Delta x(f(2)+f(4)+f(6)+f(8)+f(10))$

$= 2(3 + 4\frac{1}{3} + 5\frac{1}{3} + 6\frac{1}{4} + 7)$

$= 2(25\frac{11}{12}) = 51\frac{5}{6}$

$= 51.833\overline{3}$



b) now let $n=10$

$L_{10} = 1(f(0)+f(1)+f(2)+\dots+f(9))$

$= (1 + 2 + 3 + 3\frac{2}{3} + 4\frac{1}{3} + 4\frac{3}{4} + 5\frac{1}{3} + 5\frac{2}{3} + 6\frac{1}{4} + 6\frac{1}{2})$

$= 42.5$

$U_{10} = 1(f(1)+f(2)+\dots+f(10))$

$= (2 + 3 + 3\frac{2}{3} + 4\frac{1}{3} + 4\frac{3}{4} + 5\frac{1}{3} + 5\frac{2}{3} + 6\frac{1}{4} + 6\frac{1}{2} + 7)$

$= 48.5$

2. a) $x=0$ to $x=12$ with $n=6 \Rightarrow \Delta x = 2$

(i) $L_6 = \Delta x(f(0)+f(2)+f(4)+f(6)+f(8)+f(10))$

$= 2(9 + 8.75 + 8.25 + 7.25 + 6 + 4)$

$= 86.5$

(ii) $R_6 = \Delta x(f(2)+f(4)+f(6)+f(8)+f(10)+f(12))$

$= 2(8.75 + 8.25 + 7.25 + 6 + 4 + 1)$

$= 70.5$

(iii) $M_6 = \Delta x(f(1)+f(3)+f(5)+f(7)+f(9)+f(11))$

$= 2[8\frac{7}{8} + 8.5 + 7.75 + 6.5 + 5 + 2.75)$

$= 78.75$

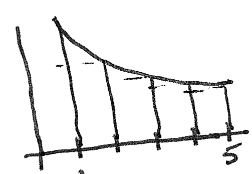
b) L_6 is an overestimate

c) R_6 is an underestimate

d) M_6 gives the best estimate b/c it escentially splits the difference w/ R_6 & L_6

3. $f(x) = \frac{1}{x}$, $[1,5]$, $n=4$

a)

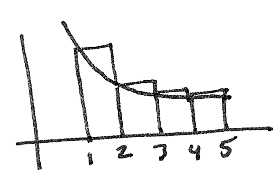


under-estimate

$R_4 = \Delta x(f(2)+f(3)+f(4)+f(5))$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} = 1.28\overline{3}$

b)



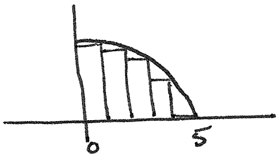
over-estimate

$L_4 = \Delta x(f(1)+f(2)+f(3)+f(4))$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12} = 2.08\overline{3}$

4. $f(x) = 25 - x^2$, $[0,5]$, $n=5$

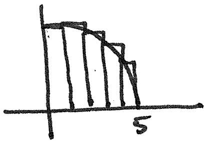
a)



under estimate

$$R_5 = \Delta x(f(1)+f(2)+f(3)+f(4)+f(5))$$
$$= 1(24+21+16+9+0)$$
$$= 70$$

b)



over estimate

$$L_5 = \Delta x(f(0)+f(1)+f(2)+f(3)+f(4))$$
$$= 1(25+24+21+16+9)$$
$$= 95$$

20. $y = x^3$, $[0,1]$

a) use def $\boxed{2}$

$$A = \lim_{n\to\infty} \Delta x f(x_1) + \Delta x f(x_2) + \cdots + \Delta x f(x_n)$$
$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$
$$x_i = a + \Delta x i = 0 + \frac{1}{n} i = \frac{i}{n}$$
$$A = \lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{n}\right)^3 + \frac{1}{n}\left(\frac{2}{n}\right)^3 + \cdots + \frac{1}{n}\left(\frac{n}{n}\right)^3$$

b) $$A = \lim_{n\to\infty} \frac{1}{n^4}(1^3+2^3+\cdots+n^3)$$
$$= \lim_{n\to\infty} \frac{1}{n^4}\left[\frac{n(n+1)}{2}\right]^2$$
$$= \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2} = \frac{1}{4}$$